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Deriving the OLS estimator with geometry

**Background**

Geometry of OLS

The OLS estimator finds the vector on the column space of that is closest to the vector.

For the following matrix,

all possible linear combinations of the columns of ,

are contained in the column space of .

The OLS estimator,

finds the linear combination of the columns of ,

(columns of multiplied by the OLS estimates)

which is the vector ,

that is as close to the vector,

as possible.

Visualising the column space

To visualise the column space of , consider a very simple example.

The following matrix,

contains 3 observations (because there are 3 rows), and 2 columns.

This matrix produces a 2-dimensional column space inside of a 3-dimensional world (note: the dimensions of the column space do not necessarily equal to the number of columns in the matrix). Since the column space is characterised by the columns of a matrix, the column space of is characterised by the following column vectors,

which produces a 2-dimensional space (which is infinitely spanning).

If columns of were linearly dependent, then the dimensions of the column space will not equal to the number of columns in .

To summarise, the space spanned by the columns in is the column space of .

These column vectors have coordinates,

- 1st column of the matrix is a vector to the coordinates of the axis

- 2nd column of the matrix is a vector to the coordinates of the axis

Therefore, if the origin is the corner of a room then the column space of

is the floor and this space spans infinitely.

All possible linear combinations of the columns of lie on the column space of . The OLS estimator is a formula,

that chooses the linear combination of the columns of , , that is as close to the vector as possible

(The vector does not necessarily lie on the column space of . If it did, then the OLS estimator would choose equal to.)

For to be as close to as possible, the length of the residual vector ,

must be as small as possible and this occurs only when is perpendicular to the column space of .

When is perpendicular to the column space of ,

becomes the orthogonal projection of onto the column space of .

When finding the linear combination of the columns of that is closest to the vector, we will have derived the OLS estimator,

Extra

If the vector lies on the column space of , then the OLS estimator will find a linear combination of the columns of that is equal to the vector and as such, the least squares residual vector will be a vector of 0s.